

等 別：三等考試  
類 科：經建行政、農業行政  
科 目：統計學  
考試時間：2 小時

座號：\_\_\_\_\_

※注意：(一)可以使用電子計算器，使用電子計算器計算之試題，須詳列解答過程。  
(二)不必抄題，作答時請將試題題號及答案依照順序寫在試卷上，於本試題上作答者，不予計分。  
(三)本科目除專門名詞或數理公式外，應使用本國文字作答。

一、某研究擬測試三種品牌汽油的每加侖所能行駛之英哩數。由於不同廠牌汽車的汽油性能特點不同，實驗選取 4 個廠牌汽車作為區集；意即，每個廠牌的汽車都用每種類型的汽油進行測試。實驗結果如下表：(以英哩/加侖為單位)

|    |   | 汽油品牌 |    |     |
|----|---|------|----|-----|
|    |   | I    | II | III |
| 汽車 | 甲 | 18   | 21 | 27  |
|    | 乙 | 24   | 23 | 28  |
|    | 丙 | 20   | 25 | 30  |
|    | 丁 | 22   | 23 | 27  |

- (一)試計算並列出適合此實驗設計的變異數分析表(ANOVA Table)。(20 分)  
(二)試寫出檢驗三種品牌汽油之每加侖英哩數差異的虛無假設與對立假設，計算檢定統計量，並說明在 5% 之顯著水準下，三種品牌汽油之每加侖英哩數是否有顯著差異。(10 分)

二、假設  $X_i$  為解釋變數、 $Y_i$  為反應變數，且  $i=1,2,\dots,n$ 。為了描述兩變數之關聯性，擬建立簡單線性迴歸模型： $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ ，其中  $\varepsilon_1, \dots, \varepsilon_n$  為獨立且相同  $N(0, \sigma^2)$  分佈之誤差。請利用下方訊息回答問題：

$$n=14, \sum_{i=1}^n X_i = 630, \sum_{i=1}^n Y_i = 520, \sum_{i=1}^n X_i Y_i = 20940, \\ \sum_{i=1}^n X_i^2 = 30300, \sum_{i=1}^n Y_i^2 = 22482。$$

- (一)試計算並寫出最小平方估計方程式。(10 分)  
(二)試計算並解釋判定係數 (Coefficient of Determination;  $R^2$ ) 及樣本相關係數 (Sample Correlation Coefficient)。(10 分)  
(三)試計算並解釋當  $X = 45$  時， $Y$  的期望值之 95% 信賴區間。(10 分)  
(參考值： $t_{0.025,12} = 2.179, t_{0.05,12} = 1.782, t_{0.025,13} = 2.160, t_{0.05,13} = 1.771$   
 $t_{0.025,14} = 2.145, t_{0.05,14} = 1.761$ )

三、定義  $X$  及  $Y$  為獨立的標準常態分布隨機變數，令  $U = X/Y$  及  $V = |Y|$ 。試計算  $U$  和  $V$  之機率密度函數，並說明  $U$  和  $V$  分別為何分布。(20分)

四、定義母群體平均值為  $\mu$ ，欲進行假設檢定問題：

$H_0$  (虛無假設) :  $\mu = 10$  ;  $H_a$  (對立假設) :  $\mu \neq 10$  。

已知母群體標準差為 6，樣本數是 36。在 95% 的信心水準之下，當真實的母群體平均值為 10.68 時，試計算並說明進行此檢定最合適的檢定統計量之檢定力 (Power) 是多少？(20分)

附表一、標準常態分布表(左尾機率)，例如： $P(Z \leq 1.96) = 0.975$

| z   | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| .0  | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1  | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2  | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3  | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4  | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5  | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6  | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7  | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8  | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9  | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |

附表二、F分布表(右尾機率)，例如： $P(F_{1,2} > 8.53) = 0.10$

| Denominator Degrees of Freedom | Area in Upper Tail | Numerator Degrees of Freedom |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
|--------------------------------|--------------------|------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|                                |                    | 1                            | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9       | 10      | 15      | 20      | 25      | 30      | 40      | 60      | 100     | 1000    |
| 1                              | .10                | 39.86                        | 49.50   | 53.59   | 55.83   | 57.24   | 58.20   | 58.91   | 59.44   | 59.86   | 60.19   | 61.22   | 61.74   | 62.05   | 62.26   | 62.53   | 62.79   | 63.01   | 63.30   |
|                                | .05                | 161.45                       | 199.50  | 215.71  | 224.58  | 230.16  | 233.99  | 236.77  | 238.88  | 240.54  | 241.88  | 245.95  | 248.02  | 249.26  | 250.10  | 251.14  | 252.20  | 253.04  | 254.19  |
|                                | .025               | 647.79                       | 799.48  | 864.15  | 899.60  | 921.83  | 937.11  | 948.20  | 956.64  | 963.28  | 968.63  | 984.87  | 993.08  | 998.09  | 1001.40 | 1005.60 | 1009.79 | 1013.16 | 1017.76 |
|                                | .01                | 4052.18                      | 4999.34 | 5403.53 | 5624.26 | 5763.96 | 5858.95 | 5928.33 | 5980.95 | 6022.40 | 6055.93 | 6156.97 | 6208.66 | 6239.86 | 6260.35 | 6286.43 | 6312.97 | 6333.92 | 6362.80 |
| 2                              | .10                | 8.53                         | 9.00    | 9.16    | 9.24    | 9.29    | 9.33    | 9.35    | 9.37    | 9.38    | 9.39    | 9.42    | 9.44    | 9.45    | 9.46    | 9.47    | 9.47    | 9.48    | 9.49    |
|                                | .05                | 18.51                        | 19.00   | 19.16   | 19.25   | 19.30   | 19.33   | 19.35   | 19.37   | 19.38   | 19.40   | 19.43   | 19.45   | 19.46   | 19.46   | 19.47   | 19.48   | 19.49   | 19.49   |
|                                | .025               | 38.51                        | 39.00   | 39.17   | 39.25   | 39.30   | 39.33   | 39.36   | 39.37   | 39.39   | 39.40   | 39.43   | 39.45   | 39.46   | 39.46   | 39.47   | 39.48   | 39.49   | 39.50   |
|                                | .01                | 98.50                        | 99.00   | 99.16   | 99.25   | 99.30   | 99.33   | 99.36   | 99.38   | 99.39   | 99.40   | 99.43   | 99.45   | 99.46   | 99.47   | 99.48   | 99.48   | 99.49   | 99.50   |
| 3                              | .10                | 5.54                         | 5.46    | 5.39    | 5.34    | 5.31    | 5.28    | 5.27    | 5.25    | 5.24    | 5.23    | 5.20    | 5.18    | 5.17    | 5.17    | 5.16    | 5.15    | 5.14    | 5.13    |
|                                | .05                | 10.13                        | 9.55    | 9.28    | 9.12    | 9.01    | 8.94    | 8.89    | 8.85    | 8.81    | 8.79    | 8.70    | 8.66    | 8.63    | 8.62    | 8.59    | 8.57    | 8.55    | 8.53    |
|                                | .025               | 17.44                        | 16.04   | 15.44   | 15.10   | 14.88   | 14.73   | 14.62   | 14.54   | 14.47   | 14.42   | 14.25   | 14.17   | 14.12   | 14.08   | 14.04   | 13.99   | 13.96   | 13.91   |
|                                | .01                | 34.12                        | 30.82   | 29.46   | 28.71   | 28.24   | 27.91   | 27.67   | 27.49   | 27.34   | 27.23   | 26.87   | 26.69   | 26.58   | 26.50   | 26.41   | 26.32   | 26.24   | 26.14   |
| 4                              | .10                | 4.54                         | 4.32    | 4.19    | 4.11    | 4.05    | 4.01    | 3.98    | 3.95    | 3.94    | 3.92    | 3.87    | 3.84    | 3.83    | 3.82    | 3.80    | 3.79    | 3.78    | 3.76    |
|                                | .05                | 7.71                         | 6.94    | 6.59    | 6.39    | 6.26    | 6.16    | 6.09    | 6.04    | 6.00    | 5.96    | 5.86    | 5.80    | 5.77    | 5.75    | 5.72    | 5.69    | 5.66    | 5.63    |
|                                | .025               | 12.22                        | 10.65   | 9.98    | 9.60    | 9.36    | 9.20    | 9.07    | 8.98    | 8.90    | 8.84    | 8.66    | 8.56    | 8.50    | 8.46    | 8.41    | 8.36    | 8.32    | 8.26    |
|                                | .01                | 21.20                        | 18.00   | 16.69   | 15.98   | 15.52   | 15.21   | 14.98   | 14.80   | 14.66   | 14.55   | 14.20   | 14.02   | 13.91   | 13.84   | 13.75   | 13.65   | 13.58   | 13.47   |
| 5                              | .10                | 4.06                         | 3.78    | 3.62    | 3.52    | 3.45    | 3.40    | 3.37    | 3.34    | 3.32    | 3.30    | 3.324   | 3.21    | 3.19    | 3.17    | 3.16    | 3.14    | 3.13    | 3.11    |
|                                | .05                | 6.61                         | 5.79    | 5.41    | 5.19    | 5.05    | 4.95    | 4.88    | 4.82    | 4.77    | 4.74    | 4.62    | 4.56    | 4.52    | 4.50    | 4.46    | 4.43    | 4.41    | 4.37    |
|                                | .025               | 10.01                        | 8.43    | 7.76    | 7.39    | 7.15    | 6.98    | 6.85    | 6.76    | 6.68    | 6.62    | 6.43    | 6.33    | 6.27    | 6.23    | 6.18    | 6.12    | 6.08    | 6.02    |
|                                | .01                | 16.26                        | 13.27   | 12.06   | 11.39   | 10.97   | 10.67   | 10.46   | 10.29   | 10.16   | 10.05   | 9.72    | 9.55    | 9.45    | 9.38    | 9.29    | 9.20    | 9.13    | 9.03    |
| 6                              | .10                | 3.78                         | 3.46    | 3.29    | 3.18    | 3.11    | 3.05    | 3.01    | 2.98    | 2.96    | 2.94    | 2.87    | 2.84    | 2.81    | 2.80    | 2.78    | 2.76    | 2.75    | 2.72    |
|                                | .05                | 5.99                         | 5.14    | 4.76    | 4.53    | 4.39    | 4.28    | 4.21    | 4.15    | 4.10    | 4.06    | 3.94    | 3.87    | 3.83    | 3.81    | 3.77    | 3.74    | 3.71    | 3.67    |
|                                | .025               | 8.81                         | 7.26    | 6.60    | 6.23    | 5.99    | 5.82    | 5.70    | 5.60    | 5.52    | 5.46    | 5.27    | 5.17    | 5.11    | 5.07    | 5.01    | 4.96    | 4.92    | 4.86    |
|                                | .01                | 13.75                        | 10.92   | 9.78    | 9.15    | 8.75    | 8.47    | 8.26    | 8.10    | 7.98    | 7.87    | 7.56    | 7.40    | 7.30    | 7.23    | 7.14    | 7.06    | 6.99    | 6.89    |
| 7                              | .10                | 3.59                         | 3.26    | 3.07    | 2.96    | 2.88    | 2.83    | 2.78    | 2.75    | 2.72    | 2.70    | 2.63    | 2.59    | 2.57    | 2.56    | 2.54    | 2.51    | 2.50    | 2.47    |
|                                | .05                | 5.59                         | 4.74    | 4.35    | 4.12    | 3.97    | 3.87    | 3.79    | 3.73    | 3.68    | 3.64    | 3.51    | 3.44    | 3.40    | 3.38    | 3.34    | 3.30    | 3.27    | 3.23    |
|                                | .025               | 8.07                         | 6.54    | 5.89    | 5.52    | 5.29    | 5.12    | 4.99    | 4.90    | 4.82    | 4.76    | 4.57    | 4.47    | 4.40    | 4.36    | 4.31    | 4.25    | 4.21    | 4.15    |
|                                | .01                | 12.25                        | 9.55    | 8.45    | 7.85    | 7.46    | 7.19    | 6.99    | 6.84    | 6.72    | 6.62    | 6.31    | 6.16    | 6.06    | 5.99    | 5.91    | 5.82    | 5.75    | 5.66    |
| 8                              | .10                | 3.46                         | 3.11    | 2.92    | 2.81    | 2.73    | 2.67    | 2.62    | 2.59    | 2.56    | 2.54    | 2.46    | 2.42    | 2.40    | 2.38    | 2.36    | 2.34    | 2.32    | 2.30    |
|                                | .05                | 5.32                         | 4.46    | 4.07    | 3.84    | 3.69    | 3.58    | 3.50    | 3.44    | 3.39    | 3.35    | 3.22    | 3.15    | 3.11    | 3.08    | 3.04    | 3.01    | 2.97    | 2.93    |
|                                | .025               | 7.57                         | 6.06    | 5.42    | 5.05    | 4.82    | 4.65    | 4.53    | 4.43    | 4.36    | 4.30    | 4.10    | 4.00    | 3.94    | 3.89    | 3.84    | 3.78    | 3.74    | 3.68    |
|                                | .01                | 11.26                        | 8.65    | 7.59    | 7.01    | 6.63    | 6.37    | 6.18    | 6.03    | 5.91    | 5.81    | 5.52    | 5.36    | 5.26    | 5.20    | 5.12    | 5.03    | 4.96    | 4.87    |
| 9                              | .10                | 3.36                         | 3.01    | 2.81    | 2.69    | 2.61    | 2.55    | 2.51    | 2.47    | 2.44    | 2.42    | 2.34    | 2.30    | 2.27    | 2.25    | 2.23    | 2.21    | 2.19    | 2.16    |
|                                | .05                | 5.12                         | 4.26    | 3.86    | 3.63    | 3.48    | 3.37    | 3.29    | 3.23    | 3.18    | 3.14    | 3.01    | 2.94    | 2.89    | 2.86    | 2.83    | 2.79    | 2.76    | 2.71    |
|                                | .025               | 7.21                         | 5.71    | 5.08    | 4.72    | 4.48    | 4.32    | 4.20    | 4.10    | 4.03    | 3.96    | 3.77    | 3.67    | 3.60    | 3.56    | 3.51    | 3.45    | 3.40    | 3.34    |
|                                | .01                | 10.56                        | 8.02    | 6.99    | 6.42    | 6.06    | 5.80    | 5.61    | 5.47    | 5.35    | 5.26    | 4.96    | 4.81    | 4.71    | 4.65    | 4.57    | 4.48    | 4.41    | 4.32    |
| 10                             | .10                | 3.29                         | 2.92    | 2.73    | 2.61    | 2.52    | 2.46    | 2.41    | 2.38    | 2.35    | 2.32    | 2.24    | 2.20    | 2.17    | 2.16    | 2.13    | 2.11    | 2.09    | 2.06    |
|                                | .05                | 4.96                         | 4.10    | 3.71    | 3.48    | 3.33    | 3.22    | 3.14    | 3.07    | 3.02    | 2.98    | 2.85    | 2.77    | 2.73    | 2.70    | 2.66    | 2.62    | 2.59    | 2.54    |
|                                | .025               | 6.94                         | 5.46    | 4.83    | 4.47    | 4.24    | 4.07    | 3.95    | 3.85    | 3.78    | 3.72    | 3.52    | 3.42    | 3.35    | 3.31    | 3.26    | 3.20    | 3.15    | 3.09    |
|                                | .01                | 10.04                        | 7.56    | 6.55    | 5.99    | 5.64    | 5.39    | 5.20    | 5.06    | 4.94    | 4.85    | 4.56    | 4.41    | 4.31    | 4.25    | 4.17    | 4.08    | 4.01    | 3.92    |
| 11                             | .10                | 3.23                         | 2.86    | 2.66    | 2.54    | 2.45    | 2.39    | 2.34    | 2.30    | 2.27    | 2.25    | 2.17    | 2.12    | 2.10    | 2.08    | 2.05    | 2.03    | 2.01    | 1.98    |
|                                | .05                | 4.84                         | 3.98    | 3.59    | 3.36    | 3.20    | 3.09    | 3.01    | 2.95    | 2.90    | 2.85    | 2.72    | 2.65    | 2.60    | 2.57    | 2.53    | 2.49    | 2.46    | 2.41    |
|                                | .025               | 6.72                         | 5.26    | 4.63    | 4.28    | 4.04    | 3.88    | 3.76    | 3.66    | 3.59    | 3.53    | 3.33    | 3.23    | 3.16    | 3.12    | 3.06    | 3.00    | 2.96    | 2.89    |
|                                | .01                | 9.65                         | 7.21    | 6.22    | 5.67    | 5.32    | 5.07    | 4.89    | 4.74    | 4.63    | 4.54    | 4.25    | 4.10    | 4.01    | 3.94    | 3.86    | 3.78    | 3.71    | 3.61    |
| 12                             | .10                | 3.18                         | 2.81    | 2.61    | 2.48    | 2.39    | 2.33    | 2.28    | 2.24    | 2.21    | 2.19    | 2.10    | 2.06    | 2.03    | 2.01    | 1.99    | 1.96    | 1.94    | 1.91    |
|                                | .05                | 4.75                         | 3.89    | 3.49    | 3.26    | 3.11    | 3.00    | 2.91    | 2.85    | 2.80    | 2.75    | 2.62    | 2.54    | 2.50    | 2.47    | 2.43    | 2.38    | 2.35    | 2.30    |
|                                | .025               | 6.55                         | 5.10    | 4.47    | 4.12    | 3.89    | 3.73    | 3.61    | 3.51    | 3.44    | 3.37    | 3.18    | 3.07    | 3.01    | 2.96    | 2.91    | 2.85    | 2.80    | 2.73    |
|                                | .01                | 9.33                         | 6.93    | 5.95    | 5.41    | 5.06    | 4.82    | 4.64    | 4.50    | 4.39    | 4.30    | 4.01    | 3.86    | 3.76    | 3.70    | 3.62    | 3.54    | 3.47    | 3.37    |